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| **Computing**

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*Computer Science*

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# Tutorial 3

*CS2109s TG35,36*

- 1 Adversarial Search in Tic-Tac-Toe
- 2 Minimax
- 3 Nonogram
- 4 Alpha-Beta Pruning



# Section 1: **Adversarial Search in Tic-Tac-Toe**



Tic-Tac-Toe - Use the minimax to determine the first move of the player.

$$Eval(n) = P(n) - O(n), \quad \text{where } P(n), O(n) \text{ are the no. of winning lines}$$

## Recap

- 1 What is the **MINIMAX** algorithm? Why is it used?
- 2 What are the ingredients needed to setup a minimax problem?

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» Actors, Actions, Leaf Costs
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- 4 [©] When was **MINIMAX** famously used in AI?

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  - » IBM Deep Blue versus Garry Kasparov in Chess.

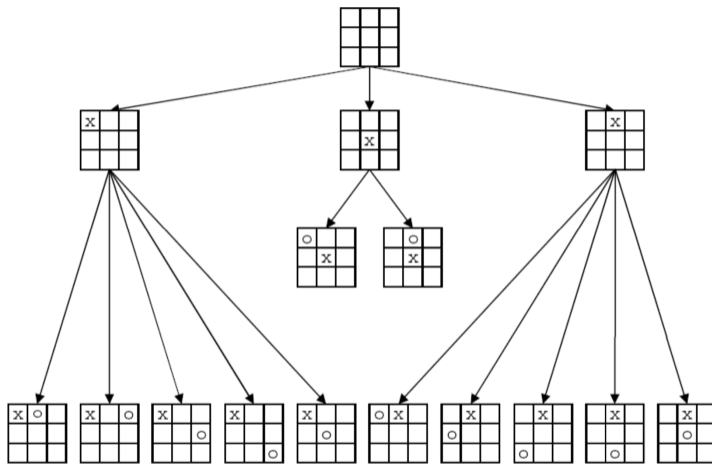


Figure 1: What is the move of the player?

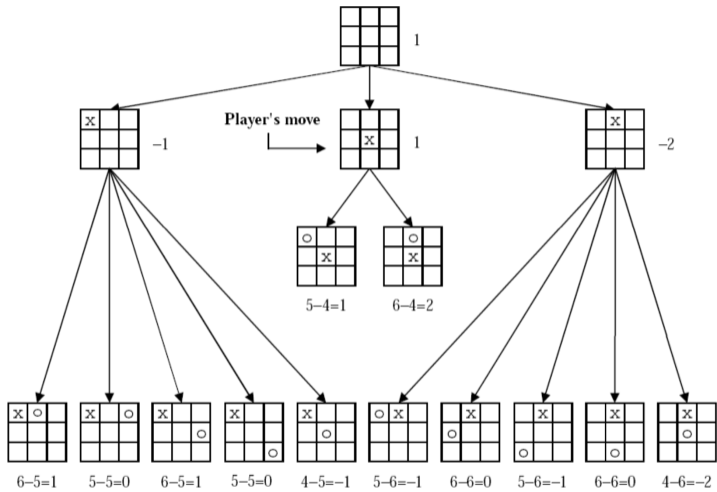


Figure 2: First move 2-ply deep search space



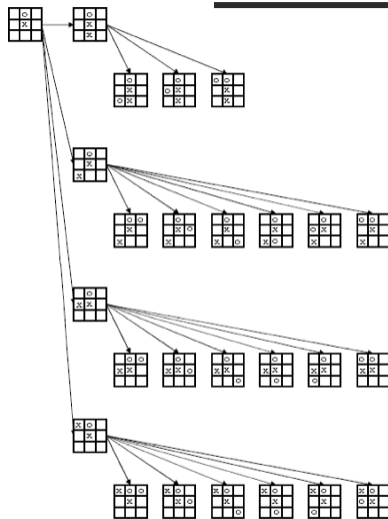


Figure 3: What is the move of the player?

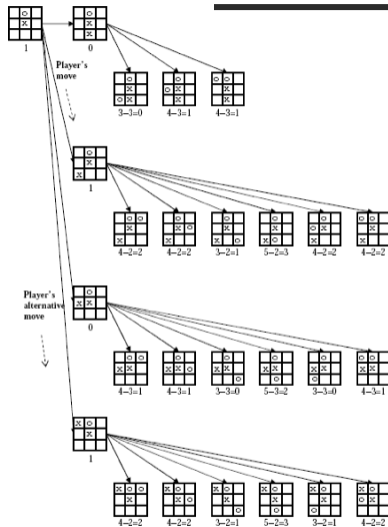


Figure 4: Second move 2-ply deep search space solution



## Section 2: **Minimax**

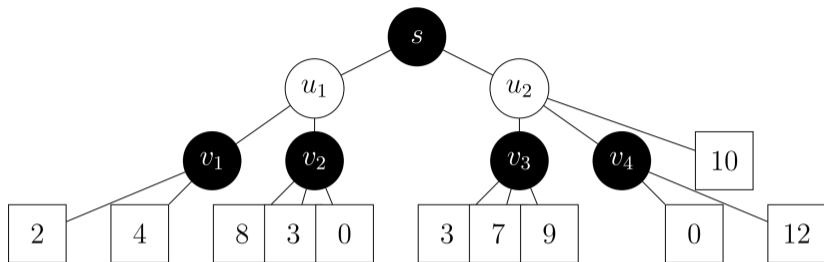


Figure 5: Alpha-Beta Tree

Run through the  $\alpha$ - $\beta$ :

- a. Right to Left
- b. Left to Right

Then determine if the effectiveness of pruning depends on iteration order.

### Recap

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- 2 What kind of efficiency do you gain?

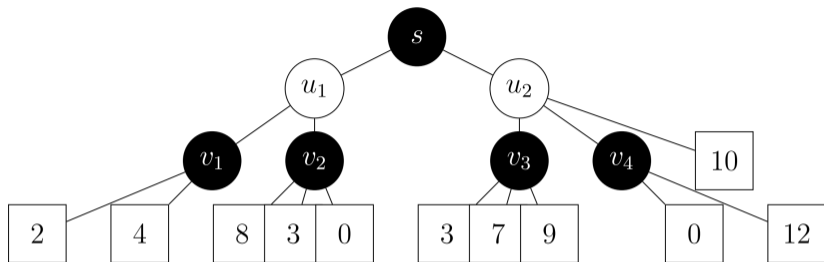


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### Recap

- 1 What does  $\alpha$ - $\beta$  do?
- 2 What kind of efficiency do you gain?
  - » Static evaluation and move generation.
- 3 What is deep cutoff?

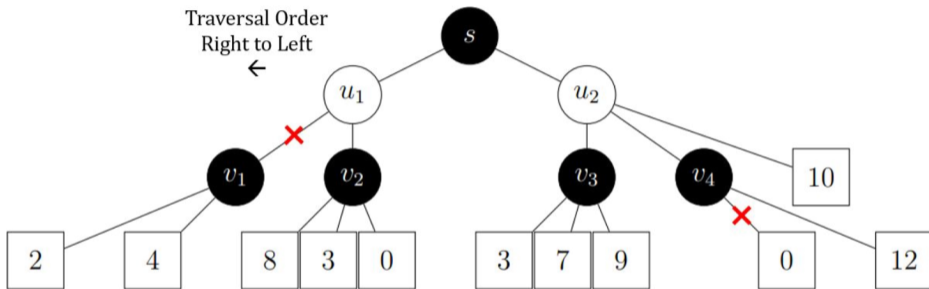


Figure 6: Right to left

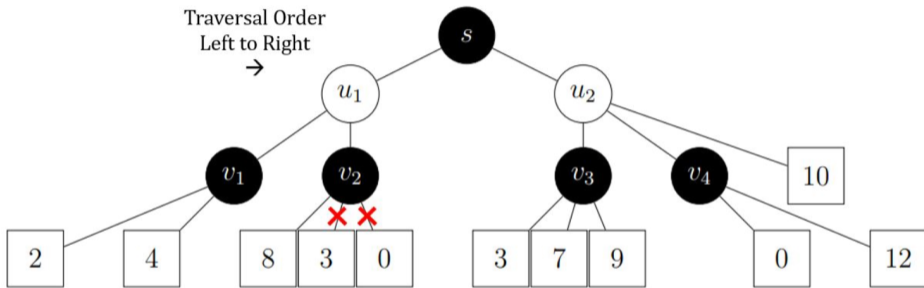


Figure 7: Left to right



## Section 3: **Nonogram**





Nonogram, aka Paint by Numbers, is a puzzle where cells are colored or left blank according to the numbers at the side of the grid.

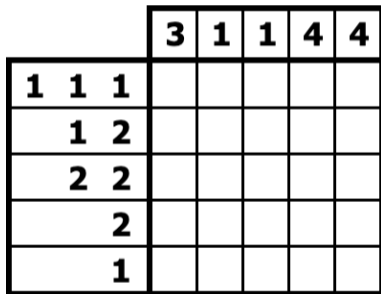


Figure 8: Initial

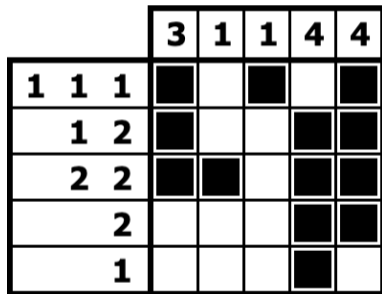


Figure 9: Solved

- 1 What are the ingredients needed for informed search?
- 2 What are the ingredients needed for local search?
- 3 What are the objectives for informed/local search?

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## Summary

**Un/Informed Search (Path):** State space, Initial, Final, Action, Transition

- › Uninformed: BFS, UCS, DFS
- › Informed: GBFS, A\*

**Local Search (Goal):** Initial state, Transition, Heuristic/Stopping criteria

- › Hill Climbing, Sim. Annealing, Beam, Genetic...

**Adversarial Search:** Actors, Actions, Leaf Costs

- › Minimax, Alpha-Beta

Having learnt both informed search and local search, you think that local search is more suitable for this problem. Give 2 possible reasons why informed search might be a bad idea.

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### Answer

- › We are only interested in the final solution.
- › Search space is large  $O(2^{n \times n})$  for a  $n \times n$  grid.
- › May not be solvable? In that case we can get a config that minimize violations.

Find a formulation for **Local Search**.

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### Answer

$n \times n$  boolean matrix, where each element is either true (if the corresponding cell is colored) or false (if the corresponding cell is not colored).

- **Initial state** is an  $n \times n$  boolean matrix with every row having random permutations of boolean vector satisfying row constraints, while the rest of the entries are set to false.
- **Transition**: we can pick a random row and generate the list of neighbors with the corresponding row permuted satisfying row constraints.
- **Heuristic/Stopping criteria**: number of instances where the constraints on the column configurations are violated.

Local search is susceptible to local minima. Describe how you can modify your solution to combat this.



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### Answer

- Introduce random restarts by repeating local search from a random initial state
- Simulated annealing search to accept a possibly `bad` state with a probability that decays over time
- beam search to perform  $k$  hill-climbing searches in parallel.



## Section 4: **Alpha-Beta Pruning**



In order for node B to NOT be pruned, what values can node A take on?

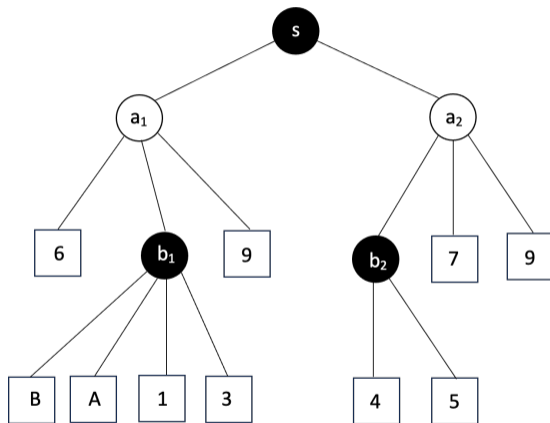


Figure 10: Find A so the B is not pruned.

```
< S -inf inf
  < a2 -inf inf
  > a2 -inf 9
  < a2 -inf 9
  > a2 -inf 7
  < a2 -inf 7
    < b2 -inf 7
    > b2 5 7
    < b2 5 7
    > b2 5 7
  > a2 -inf 5
> S 5 inf
```

```
< S 5 inf
  < a1 5 inf
  > a1 5 9
  < a1 5 9
    < b1 5 9
    > b1 5 9
    < b1 5 9
    > b1 5 9
    < b1 5 9
    > b1 Pruned val >= beta: 9 >= 9
  > a1 5 9
  < a1 5 9
  > a1 5 6
> S 6 inf
```

Pruned when  $A \geq 9$ , Not pruned when  $A \leq 8$

To help you further your understanding, not compulsory; Work for EXP!

## Tasks

- 1 Trace Manually/Use code Figure 11 to see the full capability.
  - a. Some code implemented in <https://github.com/eric-vader/CS2109s-2425s1-bonus>
- 2 How can we benefit from  $\alpha$ - $\beta$ 's efficiency?

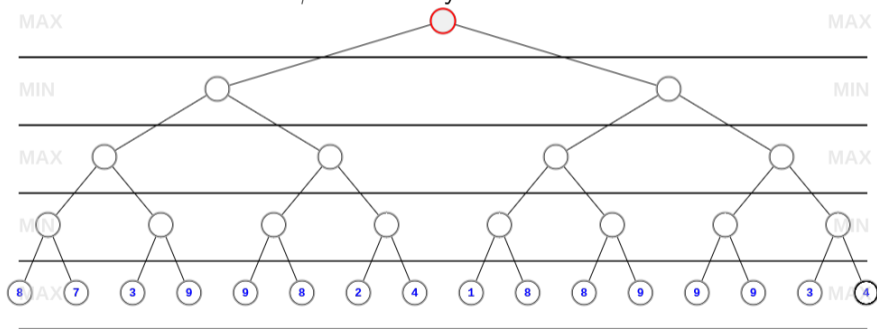


Figure 11: Alpha-Beta Example (Credit MIT)

- 1 MIT Lecture - <https://youtu.be/STjW3eH0Cik?si=YcncrXUJko5jjLzB0>
- 2 IBM Deep Blue -  
<https://www.sciencedirect.com/science/article/pii/S0004370201001291>
- 3 Game Theory Concepts Within AlphaGo -  
<https://towardsdatascience.com/game-theory-concepts-within-alphago-2443bbca36e0>
- 4 What Game Theory Reveals About Life, The Universe, and Everything -  
<https://youtu.be/mScpHTli-kM?si=CLagrjz3WVi-EkXG>

- 1 [©] and Bonus declaration is to be done here; You should show bonus to Eric.
- 2 Attempted tutorial should come with proof (sketches, workings etc...)
- 3 Random checks may be conducted.
- 4 Guest student should come and inform me.



Figure 12: Buddy Attendance: <https://forms.gle/jsGfFyfo9PTgWxib6>



